CLOSED ORBIT INSTABILITY SEEN AS A TRANSITION PROCESS DURING AND AFTER INJECTION

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I. Introduction

The general belief about the collective fields is that they don't affect the closed orbit of the beam. The comparison of the induced fields with the driving magnetic field gives many orders of magnitude difference in favor of magnets. Therefore in the stability analysis the closed orbit is assumed to be equal to zero (usually zero coincides with the symmetry axis of the vacuum chamber). Then the eigenfrequencies of all the collective modes of oscillation are searched for near the betatron sidebands of the revolution harmonics. It is shown in this paper that if the transverse impedance is large near zero frequency, although it equals zero at zero frequency, there exists a strong accumulation of the fields at the beam orbit, which leads to an enormous amplification of the closed orbit distortion during and after injection, typically within a few thousand turns. The first estimation shows that the effect could be very important for high intensity proton rings, e.g. the SNS ring at the full intensity of $2*10^{14}$ protons.

II. Basic Physics

We consider only long memory multiturn effects. Therefore the beam can be represented as one macroparticle, if we are not interested in the spread of the betatron frequencies, orbit dispersion, etc. The fields left after the beam passage can be written as a sum from previous turns of the beam transverse coordinate with the wake function. The equation for vertical (or horizontal) motion for this macroparticle is (see, e.g. [1]):

$$\ddot{y} + (\omega_b)^2 y = -\frac{N r_o c^2}{\gamma C} \sum_{k=1}^{\infty} y(s-kC) W(-kC) \qquad (1)$$

where N is the number of particles, r_0 is the particle's classical radius, W is the wake function, responsible for the collective field accumulation, C is the storage ring circumference, ω_b is the betatron frequency and the rest of the constants correspond to the standard physical notations. Let's take a constant wake function and find the solutions to equation (1). Now it reads:

$$\ddot{y} + (\omega_b)^2 y = \chi \sum_{k=1}^{k0} y(s-kC), \qquad (2)$$

with χ=-$Nr_0Wc^2/C\gamma$, and k_0 equal to the number of turns after injection. The equation has three (!) eigenfrequencies. If we assume that the collective field tuneshift is small compared to the betatron frequency, these three frequencies are (we assume $exp(-i\omega t)$ dependence on time): $\omega_1=\omega_b$-$exp(i2\pi\nu_b)\chi/2\omega_b(1$-$exp(i2\pi\nu_b))$, $\omega_2=c.c.\omega_1$, $\omega_3=+i\chi\omega_0/2\pi\omega_b^2$. The first two values are just well known solutions around betatron frequency, being stable since χ is usually positive. The last eignefrequency is close to zero, which shows that the centroid position of the beam starts to drift from the center exponentially (again for the case of the positive χ). Since this eigenfrequency appears near closed orbit zero frequency and the centroid doesn't execute betatron oscillations, we call it instability of the closed orbit.

A strange feature of the increment of this instability is that it is inversely proportional to the square (!) of the betatron frequency and doesn't depend on the fractional part of the betatron tune (the first two solutions do depend on the fractional betatron tune). But this is valid only when the wake function is uniformly distributed around the ring. The remarkable feature of this instability is that it strongly depends on how the wake is localized in the ring. Let's take another extreme case when the wake is localized at one point (its length is much less than the beta function at this point). The equation (2) should be replaced with the difference equation (let's write it in normalized variables and assume the transverse angle kick is $-\dfrac{Nr_oW}{\gamma}\sum_{k=1}^{\infty}y(s-kC)$):

$$\begin{aligned} y_n &= \cos(\mu)y_{n-1}+\sin(\mu)p_{n-1}, \\ p_n &= -\sin(\mu)y_{n-1}+\cos(\mu)p_{n-1}+\kappa\beta\, s_{n-1}, \\ s_n &= s_{n-1}+y_{n-1}, \end{aligned} \qquad (3)$$

where κ =-Nr_0W/γ, β is the beta function at the point of the wake, s stands for the sum of coordinates at this point. The matrix eigenvalues are:

$$\lambda_{1,2}=exp(\pm i2\pi\nu_b)\pm i\kappa\beta/2(1-exp(\pm i2\pi\nu_b)),$$

$$\lambda_3=+1+\kappa\beta cos(\pi\nu_b)/2\ sin(\pi\nu_b), \qquad (4)$$

$$\textit{the increment } \delta=+\kappa\beta cos(\pi\nu_b)/2\ sin(\pi\nu_b).$$

Now the third eigenvalue, which corresponds to the instability of the closed orbit, depends on the beta function at the wake point, and on the fractional part of the betatron tune (contrary to the distributed wake above), and the instability happens above integer (for positive fractional tune) when the betatron oscillations are stable. This opposite case shows unique properties of the instability – its increments (decrements) strongly depend on the impedance distributions over the ring unlike the conventional betatron instabilities, which (if far from integer and half integer resonances) depend only on integral values of the impedance.

II. Practical cases – resistive wall and "surrounded" resistive wall wakes.

In this section we present two practical cases – the resistive wall case for the thick vacuum chamber and the thin vacuum chamber surrounded by magnets. For the first case the wake function is equal to:

$$W(z) = -\frac{2}{\pi b^3}\sqrt{\frac{c}{\sigma}}\frac{C}{|z|^{1/2}}, \qquad (5)$$

where we take conductivity σ in CGS system (in our case, σ=1.3$10^{16}$s^{-1} for the stainless steel), C again is the circumference, the length units should be taken in the same system as beta-function and the particle classical radius. This wake function is valid only when the fields don't penetrate the vacuum chamber. But in our situation we are hunting for the low frequency (1 kHz) effects, and the skin depth for the stainless steel is already big (about 6 mm) compared to thickness of the most vacuum chambers (usually, their thickness of the order of 1 mm). Therefore we should take into account pieces, which surround the vacuum pipe. Figure 1 shows a simplified scheme of the magnet frame (on the left) with the chamber inside. This geometry is not suitable for analytic calculations. For the estimation it can be replaced by the azimuthally symmetric geometry (on the right), with the round vacuum chamber surrounded by a round piece of metal with very high permeability (of the order of 1000). As is shown in Ref.[2], when the outer layer has much higher surface impedance (restive wall impedance of the surrounding metal is approximately $\sqrt{\mu}$ higher), then the whole image current flows through the chamber, and its transverse impedance can be estimated (from Panofsky-Wenzel formula) as $Z_\perp(\omega) = \frac{2c}{\omega\, b^2} R$, where R is just the resistance of the layer. The wake function can be obtained by taking the Fourier transform of this expression:

$$W(z) = -\frac{i2cR}{2\pi b^2}\int_{-\infty}^{\infty}\frac{d\omega}{\omega}\exp(i\omega\frac{z}{c}) = -\frac{2cR}{b^2}. \qquad (6)$$

In order to match the units with (1), the resistance R should be taken in CGS units, namely

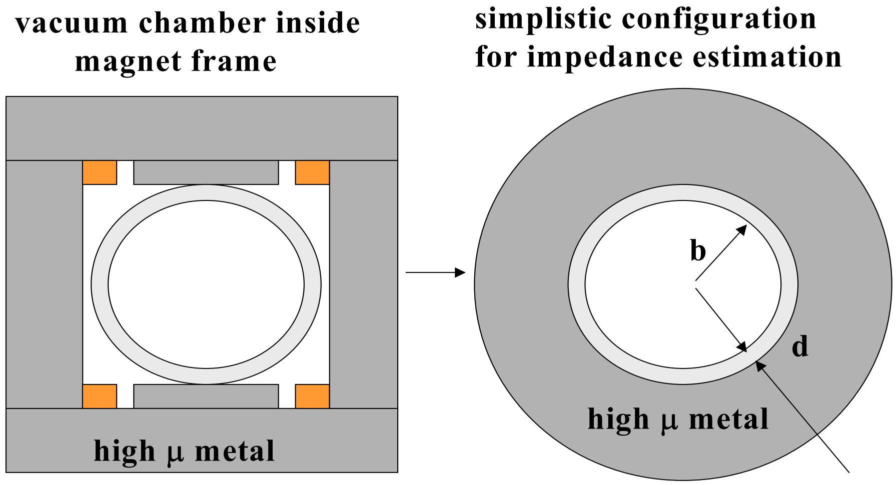

Figure 1 Simplified vacuum chamber model.

equal to $R=L/(2\pi b\ d\ \sigma)$, with $\sigma=1.310^{16}s^{-1}$ for stainless steel.

ESTIMATION FOR THE SNS RING

Consider the injection kickers, which have ceramic chambers coated with TiN. For estimation we use the following numbers: $\sigma=2.210^{16}s^{-1}$, L=5m (the total length), $d=9\ 10^{-6}$ m (the thickness), b=0.08 m, $N=2*10^{14}$ (total number of protons), β = 7m (average beta function at the kicker position), fractional betatron tune ν_b =0.2. $W=-2cL/(2\pi b^3\ d\ \sigma)$. The instability decrement is taken from equation (4). We have the increment $\delta=(Nr_0\beta cL\cos(\pi\nu_b))/(\pi b^3\ d\ \sigma\ 2\sin(\pi\nu_b))=4.2*10^{-3}$, or the growth rate $1/\delta$ =236 turns. Therefore the initial distortion could be multiplied by order of e^4 during the 1000 turns injection. Therefore the low frequency impedance should be reduced by order of magnitude.

Distributed resistive wall effects are smaller, but the localized resistive wall due to the collimators gives almost the same integrated wake as the rest of the ring. An estimation of this effect alone gives about 12% growth of the closed orbit during 1000 turns. The strange thing about the resistive wake is that the increment of the instability depends on the squared current (one more strange property of the instability). The solution for the resistive wall wake (uniformly distributed and localized), and the more general situation of several localized impedances in the ring is given in the Appendix.

WHY TRANSITION

All the effects described work only when the fields live inside the vacuum chamber. The wake fields totally change after the fields penetrate through the vacuum chamber. Normally it happens after few thousand turns, which correspond to the frequencies below 1 kHz. Moreover, since the transverse impedance goes to zero at zero frequency, the total integral of the wake function is equal to zero, therefore the stationary orbit produces no integral effect – the effect is essentially transient, which is, probably, not important for the short pulse accumulators.

But, obviously, when the intensity is large and the rise time is shorter than the penetration time, only the wake at times shorter than the instability rise time matters and the instability will last until the beam is lost.

CONCLUSION and RECCOMMENDATIONS

The instability presented here reveals many new features, such as the strange dependence on the wake distributions over the ring, dependence on the inverse squared frequency in the distributed case, and opposite behavior to the conventional resistive wall behavior in the localized case. There is no chromaticity effect on the instability, since it is not coupled to the betatron oscillation. Therefore it is hard to provide usual damping mechanisms for it. If the low frequency (1kHz – 1MHz) integral impedance is large, it will present a big danger for storage ring operation.

The impedance of interest should be better understood and reduced if necessary (as it was shown, about a factor ten of reduction is needed in the case of thin coatings for the SNS injection kicker). The vacuum chamber elements should be measured for low frequency. Some cures (dynamic orbit correction, etc.) should be developed.

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[1] A. Chao, "Physics of collective beam instabilities in high energy accelerators", John Wiley & Sons, Inc. (1993) p 172.

[2] V. Danilov, et al. "ESTIMATION FOR THE SNS RING INJECTION MAGNETS COATING IMPEDANCE", SNS Tech Memo, August 2001

APPENDIX

The uniformly distributed resistive wake (5) produces the equation of the following kind:

$$\ddot{y} + (\omega_b)^2 y = \eta \sum_{k=1}^{k0} \frac{y(s-kC)}{\sqrt{ks}}, \quad (1\ \mathrm{a})$$

where η=$-Nr_0 2\sqrt{(c/\sigma)}\, c^2/\pi b^3\gamma$. If the instability is week, we look for the y solution in the form $y_0\exp(\delta\ n)$, where n is number of turns, $\delta$is the increment per turn. The sum in the

R.H.S. of (1 a) could be calculated using approximate formula
$\sum_{k=1}^{\infty}\frac{\exp(-\delta k)}{\sqrt{k}} \approx \int_0^{\infty}\frac{\exp(-\delta k)}{\sqrt{k}}dk = \sqrt{\frac{\pi}{\delta}}$. After straightforward algebra we get:

$$\delta = \frac{N^2 r_0^2 c C^3}{4\pi^5\gamma^2\nu_b^2\sigma b^6}. \quad (2\ a)$$

For the SNS parameters (c=250m, b=0.01 m, ν_b=6, N=2*10^{14}) it is equal about 1.8*10^{-8}, which is negligible. But the situation change when the wake is localized. In this case the approach of equation (4) doesn't work, but some analysis still possible. We analyze only the closed orbit. If we have a constant (or almost constant) angle kick at some point, the coordinate value at this point is: $y = \frac{\Delta y'\beta\cos(\pi\nu_b)}{2\sin(\pi\nu_b)}$. The kick is equal to
$\frac{2Nr_o L_{wake}}{\pi b^3\gamma}\sqrt{\frac{c}{\sigma}}\sum_{k=1}^{\infty}\frac{y(s-kC)}{\sqrt{ks}}$. Again using $\sum_{k=1}^{\infty}\frac{\exp(-\delta k)}{\sqrt{k}} \approx \int_0^{\infty}\frac{\exp(-\delta k)}{\sqrt{k}}dk = \sqrt{\frac{\pi}{\delta}}$ relation, the increment per turn is

$$\delta = sign(\{\nu_b\})\frac{N^2 r_0^2\beta^2 c L_{wake}^2\cos(\pi\nu_b)^2}{\pi\gamma^2 C\sigma b^6\sin(\pi\nu_b)^2}, \quad (3\ a)$$

which gives for SNS collimators (b=0.025 m, L_{wake}=10m, β=7m, $\nu_b\approx$0.1 with taking into account space charge tuneshift, with stainless steel conductivity σ=1.310^{16}s^{-1}) increment δ=1.17*10^{-4}. This gives a sizable effect of 12% orbit growth after 1000 turns, and doubles the orbit deviation after 6000 turns (if the wake function still has the same form at such big times). Of course there are many nonuniformities in the ring. Now we consider the case when we have two localized constant wakes in order to understand the behavior of instability for nonuniformly distributed wakes.

If we consider two locations with impedances, the conventional instability's increment changes according to the bulk value of the impedance at the unstable frequency. One might expect similar behavior for the closed orbit instability. But the situation is essentially different! Addition of one more wake increments the number of modes by one! The new modes could be stable or unstable depending on the phase advance between impedances. Consider two localized constant wakes. Following the previous calculations, we find the orbit deviations at points 1,2 of the localized wakes positions:

$$\begin{aligned} y_1 &= \frac{\theta_1\beta_1\cos(\pi\nu_b)}{2\sin(\pi\nu_b)} + \frac{\theta_2\sqrt{\beta_1\beta_2}\cos(\pi\nu_b-\varphi)}{2\sin(\pi\nu_b)}, \\ y_2 &= \frac{\theta_1\sqrt{\beta_1\beta_2}\cos(\pi\nu_b-\varphi)}{2\sin(\pi\nu_b)} + \frac{\theta_2\beta_2\cos(\pi\nu_b)}{2\sin(\pi\nu_b)}, \end{aligned} \quad (4\ a)$$

where $\theta_{1,2} = -\frac{Nr_o W_{1,2}}{\gamma}\sum_{k=1}^{\infty} y_{1,2}(s-kC)$, φ is the betatron phase advance between the two points. Using $\sum_{k=1}^{\infty}\exp(-\delta k) \approx \frac{1}{\delta}$ one can rewrite (4 a) as

$$
\begin{aligned}
y_1\delta &= \frac{c_1 y_1 \cos(\pi\nu_b)}{2\sin(\pi\nu_b)} + \frac{c_2 y_2 \sqrt{\beta_1/\beta_2}\cos(\pi\nu_b - \varphi)}{2\sin(\pi\nu_b)}, \\
y_2\delta &= \frac{c_1 y_1 \sqrt{\beta_2/\beta_1}\cos(\pi\nu_b - \varphi)}{2\sin(\pi\nu_b)} + \frac{c_2 y_2 \cos(\pi\nu_b)}{2\sin(\pi\nu_b)},
\end{aligned}
\qquad (5\text{ a})
$$

with $c_{1,2} = -\frac{Nr_o W_{1,2}\beta_{1,2}}{\gamma}$. There exist two eigenvalues for δ:

$$
\delta_{1,2} = -\frac{(c_1+c_2)\cot(\pi\nu_b) \pm \sqrt{(c_1-c_2)^2\cot(\pi\nu_b)^2 + 4c_1c_2\frac{\cos(\pi\nu_b-\varphi)^2}{\sin(\pi\nu_b)^2}}}{2}. \quad (6\text{ a})
$$

One can see that the sign of the increment depends on the phase advance between two points, betatron tune, beta function and wake strengths. In the general case, there exist stable and unstable harmonics, which should be determined from the integral equation of this type:

$$
y(s)\lambda = -\frac{Nr_o}{\gamma}\oint ds' \frac{W(s')y(s')\sqrt{\beta(s)\beta(s')}\cos(|\pi\nu_b - \varphi(s,s')|)}{2\sin(\pi\nu_b)}, \quad (7\text{ a})
$$

where *W(s)* denotes constant wake function per unit length and λ is the eigenvalue to be found. In general, this equation has an infinite number of roots – there should be positive and negative eigenvalues, which correspond to the stable and unstable harmonics.